ELASTICIDAD

OPTIMIZACIÓN

DERIVADA X2

ÁREA

APLICACIÓN INTEGRAL MARGINAL

SE MUESTRAN A CONTINUACIÓN LOS CÁLCULOS NECESARIOS PARA LA RESOLUCIÓN DE LOS EJERCICIOS DEL SEGUNDO PARCIAL. CIERTAS CONSIDERACIONES TALES COMO DOMINIOS, INTERPRETACIONES ECONÓMICAS, ETC. NO ESTÁN ESPECIFICADAS EN ESTE DOCUMENTO.

ELASTICIDAD: 2020 EV2L ELASTICIDAD

Calcular la elasticidad de la

curva de demanda	en	, donde	representa el número de unidades y	el precio y clasificarla
$x^2 + p^2 - 100 = 0$	$p = 8$	$0 \leq x \leq 10$	$0 \leq p \leq 10$	
$2x^2 + 2p^2 - 200 = 0$	$p = 8$	$0 \leq x \leq 10$	$0 \leq p \leq 10$	
$x^2 + p^2 - 400 = 0$	$p = 15$	$0 \leq x \leq 20$	$0 \leq p \leq 20$	
$2x^2 + 2p^2 - 800 = 0$	$p = 15$	$0 \leq x \leq 20$	$0 \leq p \leq 20$	

RESOLUCIÓN

$$\eta = \frac{p}{x} . \frac{dx}{dp}$$

$$2xdx + 2pdp = 0 \Rightarrow 2xdx = -2pdp \Rightarrow \frac{dx}{dp} = -\frac{p}{x}$$

$$\eta = \frac{p}{x} . \frac{dx}{dp} = -\frac{p}{x} . \frac{p}{x} = -\frac{p^2}{x^2} = -\frac{p^2}{100 - p^2}\bigg|_{p=8} = -\frac{16}{9} \cong -1{,}8$$

$|\eta| \cong 1{,}8 > 1$ es elástica

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$$\eta = \frac{p}{x} . \frac{dx}{dp} = -\frac{p}{x} . \frac{p}{x} = -\frac{p^2}{x^2} = -\left.\frac{p^2}{400 - p^2}\right\vert_{p=15} = -\frac{9}{7} \cong -1{,}29$$ $\lvert \eta \rvert \cong 1{,}29 > 1$ es elástica
$$\eta = \frac{p}{x} . \frac{dx}{dp}$$ $$4xdx + 4pdp = 0 \Rightarrow 4xdx = -4pdp \Rightarrow \frac{dx}{dp} = -\frac{p}{x}$$ $$\eta = \frac{p}{x} . \frac{dx}{dp} = -\frac{p}{x} . \frac{p}{x} = -\frac{p^2}{x^2} = -\left.\frac{p^2}{400 - p^2}\right\vert_{p=15} = -\frac{9}{7} \cong -1{,}29$$ $\lvert \eta \rvert \cong 1{,}29 > 1$ es elástica

OPTIMIZACIÓN: 2020 EV2L OPTIMIZACIÓN

Dada la función de demanda de una empresa

	y sabiendo que su función de costo medio es
$p=-\frac{x}{2}+100$	$\bar{C}(x)=\frac{x}{25}+\frac{420}{x}-8$
$p=-\frac{x}{3}+100$	$\bar{C}(x)=\frac{x}{15}+\frac{420}{x}-8$
$p=-\frac{x}{5}+100$	$\bar{C}(x)=\frac{x}{20}x+\frac{420}{x}-8$
$p=-\frac{x}{4}+100$	$\bar{C}(x)=\frac{x}{12}+\frac{420}{x}-8$

Hallar el nivel de producción que maximiza el beneficio de esa empresa. ¿Cuál es ese beneficio máximo?

RESOLUCIÓN

$$p=-\frac{x}{2}+100 \Rightarrow I(x)=\left(-\frac{x}{2}+100\right).x=-\frac{x^2}{2}+100x$$

$$\bar{C}(x)=\frac{x}{25}+\frac{420}{x}-8 \Rightarrow C(x)=\frac{x^2}{25}+420-8x$$

$$B(x)=-\frac{x^2}{2}+100x-\frac{x^2}{25}-420+8x=-\frac{27}{50}x^2+108x-420$$

$$DomB=[0;\ +\infty)$$

$$B'(x)=-\frac{27}{25}x+108=0 \Rightarrow x=\frac{108.25}{27}=100$$

$$B''(x)=-\frac{27}{25}<0, \forall x, \Rightarrow x=100\ es\ máximo$$

El nivel de producción que maximiza el beneficio es de 100 unidades

$$B(100)=-\frac{27}{50}100^2+108.100-420=4980$$

$$p=-\frac{x}{3}+100 \Rightarrow I(x)=\left(-\frac{x}{3}+100\right).x=-\frac{x^2}{3}+100x$$

$$\bar{C}(x)=\frac{x}{15}+\frac{420}{x}-8 \Rightarrow C(x)=\frac{x^2}{15}+420-8x$$

$$B(x)=-\frac{x^2}{3}+100x-\frac{x^2}{15}-420+8x=-\frac{2}{5}x^2+108x-420$$

$$DomB=[0;\ +\infty)$$

$$B'(x)=-\frac{4}{5}x+108=0 \Rightarrow x=135$$

$$B''(x)=-\frac{4}{5}<0, \forall x, \Rightarrow x=135\ es\ máximo$$

El nivel de producción que maximiza el beneficio es de 135 unidades

$$B(135)=6870$$

$$p=-\frac{x}{5}+100 \Rightarrow I(x)=\left(-\frac{x}{5}+100\right).x=-\frac{x^2}{5}+100x$$

$\bar{C}(x) = \frac{x}{20} + \frac{420}{x} - 8 \Rightarrow C(x) = \frac{x^2}{20} + 420 - 8x$ $B(x) = -\frac{x^2}{5} + 100x - \frac{x^2}{20} - 420 + 8x = -\frac{1}{4}x^2 + 108x - 420$ $DomB = [0;\ +\infty)$ $B'(x) = -\frac{1}{2}x + 108 = 0 \Rightarrow x = 216$ $B''(x) = -\frac{1}{2} < 0, \forall x, \Rightarrow x = 216\ es\ máximo$ El nivel de producción que maximiza el beneficio es de 216 unidades $B(216) = 11244$
$p = -\frac{x}{4} + 100 \Rightarrow I(x) = \left(-\frac{x}{4} + 100\right).x = -\frac{x^2}{4} + 100x$ $\bar{C}(x) = \frac{x}{12} + \frac{420}{x} - 8 \Rightarrow C(x) = \frac{x^2}{12} + 420 - 8x$ $B(x) = -\frac{x^2}{4} + 100x - \frac{x^2}{12} - 420 + 8x = -\frac{1}{3}x^2 + 108x - 420$ $DomB = [0;\ +\infty)$ $B'(x) = -\frac{2}{3}x + 108 = 0 \Rightarrow x = 162$ $B''(x) = -\frac{2}{3} < 0, \forall x, \Rightarrow x = 162\ es\ máximo$ El nivel de producción que maximiza el beneficio es de 162 unidades $B(162) = 8328$

DERIVADA X2: 2020 derivx2

<table>
<tr><td rowspan="2">Hallar</td><td>z_x</td><td rowspan="2">y</td><td>z_{xy}</td><td rowspan="2">para</td><td>$z = 5x.\ln(x^2 + y)$</td></tr>
<tr><td>z_y</td><td>z_{yx}</td><td>$z = 5y.\ln(x + y^2)$</td></tr>
</table>

RESUELTO

$z_x = 5.\ln(x^2 + y) + 5x.\frac{1}{x^2 + y}.2x = 5.\ln(x^2 + y) + \frac{10x^2}{x^2 + y}$ $z_{xy} = \frac{5}{x^2 + y} - \frac{10x^2}{(x^2 + y)^2}$
$z_y = 5.\ln(x + y^2) + 5y.\frac{1}{x + y^2}.2y = 5.\ln(x + y^2) + \frac{10y^2}{x + y^2}$ $z_{yx} = \frac{5}{x + y^2} - \frac{10y^2}{(x + y^2)^2}$

ÁREA: 2020 EV2L ÁREA

Hallar el área de la región encerrada por las curvas:

$2y = 4x - x^2 \qquad 2y = x - 4$
$6y = 12x - 3x^2 \qquad 6y = 3x - 12$
$\frac{1}{2}y = x - \frac{1}{4}x^2 \qquad \frac{1}{2}y = \frac{1}{4}x - 1$
$3y = 6x - \frac{3}{2}x^2 \qquad 3y = \frac{3}{2}x - 6$

RESUELTO

$4x - x^2 = x - 4 \Rightarrow 0 = x^2 - 3x - 4 \Rightarrow x = \frac{3 \pm \sqrt{9 + 16}}{2} = \frac{3 \pm 5}{2} \Rightarrow x = 4 \; o \; x = -1$ $2y = 4x - x^2 \Rightarrow y = 2x - \frac{1}{2}x^2 \Rightarrow y(0) = 0$ $2y = x - 4 \Rightarrow y = \frac{1}{2}x - 2 \Rightarrow y(0) = -2$ $A = \int_{-1}^{4} \left(2x - \frac{1}{2}x^2 - \frac{1}{2}x + 2\right) dx = \int_{-1}^{4} \left(-\frac{1}{2}x^2 + \frac{3}{2}x + 2\right) dx =$ $= \left(-\frac{1}{6}x^3 + \frac{3}{4}x^2 + 2x\right\|_{-1}^{4} = -\frac{1}{6}64 + \frac{3}{4}16 + 8 - \left(\frac{1}{6} + \frac{3}{4} - 2\right) =$

$= \frac{28}{3} + \frac{13}{12} = \frac{125}{12} \cong 10{,}42$
$12x - 3x^2 = 3x - 12 \Rightarrow 0 = 3x^2 - 9x - 12 \Rightarrow 0 = x^2 - 3x - 4 \Rightarrow$ $\Rightarrow x = \frac{3 \pm \sqrt{9+16}}{2} = \frac{3 \pm 5}{2} \Rightarrow x = 4 \ o \ x = -1$ $6y = 12x - 3x^2 \Rightarrow y = 2x - \frac{1}{2}x^2 \Rightarrow y(0) = 0$ $6y = 3x - 12 \Rightarrow y = \frac{1}{2}x - 2 \Rightarrow y(0) = -2$ $A = \int_{-1}^{4} \left(2x - \frac{1}{2}x^2 - \frac{1}{2}x + 2\right) dx = \int_{-1}^{4} \left(-\frac{1}{2}x^2 + \frac{3}{2}x + 2\right) dx =$ $= \left(-\frac{1}{6}x^3 + \frac{3}{4}x^2 + 2x\right\|_{-1}^{4} = -\frac{1}{6}64 + \frac{3}{4}16 + 8 - \left(\frac{1}{6} + \frac{3}{4} - 2\right) =$ $= \frac{28}{3} + \frac{13}{12} = \frac{125}{12} \cong 10{,}42$
$x - \frac{1}{4}x^2 = \frac{1}{4}x - 1 \Rightarrow 0 = \frac{1}{4}x^2 - \frac{3}{4}x - 1 \Rightarrow 0 = x^2 - 3x - 4 \Rightarrow$ $\Rightarrow x = \frac{3 \pm \sqrt{9+16}}{2} = \frac{3 \pm 5}{2} \Rightarrow x = 4 \ o \ x = -1$ $\frac{1}{2}y = x - \frac{1}{4}x^2 \Rightarrow y = 2x - \frac{1}{2}x^2 \Rightarrow y(0) = 0$ $\frac{1}{2}y = \frac{1}{4}x - 1 \Rightarrow y = \frac{1}{2}x - 2 \Rightarrow y(0) = -2$ $A = \int_{-1}^{4} \left(2x - \frac{1}{2}x^2 - \frac{1}{2}x + 2\right) dx = \int_{-1}^{4} \left(-\frac{1}{2}x^2 + \frac{3}{2}x + 2\right) dx =$ $= \left(-\frac{1}{6}x^3 + \frac{3}{4}x^2 + 2x\right\|_{-1}^{4} = -\frac{1}{6}64 + \frac{3}{4}16 + 8 - \left(\frac{1}{6} + \frac{3}{4} - 2\right) =$ $= \frac{28}{3} + \frac{13}{12} = \frac{125}{12} \cong 10{,}42$
$6x - \frac{3}{2}x^2 = \frac{3}{2}x - 6 \Rightarrow 0 = \frac{3}{2}x^2 - \frac{9}{2}x - 6 \Rightarrow 0 = x^2 - 3x - 4 \Rightarrow$ $\Rightarrow x = \frac{3 \pm \sqrt{9+16}}{2} = \frac{3 \pm 5}{2} \Rightarrow x = 4 \ o \ x = -1$ $3y = 6x - \frac{3}{2}x^2 \Rightarrow y = 2x - \frac{1}{2}x^2 \Rightarrow y(0) = 0$ $3y = \frac{3}{2}x - 6 \Rightarrow y = \frac{1}{2}x - 2 \Rightarrow y(0) = -2$ $A = \int_{-1}^{4} \left(2x - \frac{1}{2}x^2 - \frac{1}{2}x + 2\right) dx = \int_{-1}^{4} \left(-\frac{1}{2}x^2 + \frac{3}{2}x + 2\right) dx =$ $= \left(-\frac{1}{6}x^3 + \frac{3}{4}x^2 + 2x\right\|_{-1}^{4} = -\frac{1}{6}64 + \frac{3}{4}16 + 8 - \left(\frac{1}{6} + \frac{3}{4} - 2\right) =$ $= \frac{28}{3} + \frac{13}{12} = \frac{125}{12} \cong 10{,}42$

APLICACIÓN INTEGRAL MARGINAL: 2020 EV2L INTEGRAL MARGINAL

La función de ingreso marginal de un fabricante es

$I'(x) = \frac{2000}{\sqrt{300x}}$	$I'(x) = \frac{3000}{\sqrt{200x}}$	$I'(x) = \frac{3000}{\sqrt{300x}}$	$I'(x) = \frac{2000}{\sqrt{200x}}$

Encontrar el cambio del ingreso total del fabricante si la producción aumenta de

500 a 800 unidades	400 a 700 unidades	300 a 600 unidades	500 a 800 unidades

RESUELTO

$\int_{500}^{800} \frac{2000}{\sqrt{300x}} dx = \frac{2000}{\sqrt{300}} \int_{500}^{800} \frac{1}{\sqrt{x}} dx = \frac{2000}{\sqrt{300}}.\left(2\sqrt{x}\right|_{500}^{800} =$ $= \frac{2000}{\sqrt{300}}.\left(2\sqrt{800} - 2\sqrt{500}\right) \cong 1367{,}99$
$\int_{400}^{700} \frac{3000}{\sqrt{200x}} dx = \frac{3000}{\sqrt{200}} \int_{400}^{700} \frac{1}{\sqrt{x}} dx = \frac{3000}{\sqrt{200}}.\left(2\sqrt{x}\right|_{400}^{700} =$ $= \frac{3000}{\sqrt{200}}.\left(2\sqrt{700} - 2\sqrt{400}\right) \cong 2739{,}69$
$\int_{300}^{600} \frac{3000}{\sqrt{300x}} dx = \frac{3000}{\sqrt{300}} \int_{300}^{600} \frac{1}{\sqrt{x}} dx = \frac{3000}{\sqrt{300}}.\left(2\sqrt{x}\right|_{300}^{600} =$ $= \frac{3000}{\sqrt{300}}.\left(2\sqrt{600} - 2\sqrt{300}\right) \cong 2485{,}28$
$\int_{500}^{800} \frac{2000}{\sqrt{200x}} dx = \frac{2000}{\sqrt{200}} \int_{500}^{800} \frac{1}{\sqrt{x}} dx = \frac{2000}{\sqrt{200}}.\left(2\sqrt{x}\right|_{500}^{800} =$ $= \frac{2000}{\sqrt{200}}.\left(2\sqrt{800} - 2\sqrt{500}\right) \cong 1675{,}44$