MATEMATICA

51

TRIGONOMETRIA
Trigonometría

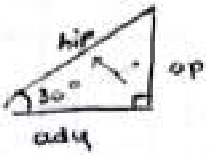
PITAGORAS

$$a^2 + b^2 = c^2$$

ej =

$$3^2 + 4^2 = c^2$$
$$9 + 16 = c^2$$
$$25 = c^2$$
$$\sqrt{25} = c$$
$$\boxed{5 = c}$$

SENO / COSENO / TANGENTE

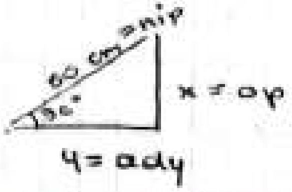


$$\text{sen } \alpha = \frac{op}{hip} \qquad \cos \alpha = \frac{ady}{hip} \qquad \tan \alpha = \frac{op}{ady}$$

$$\underbrace{\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad}_{}$$

Sirven para identificar incognitas en triangulos

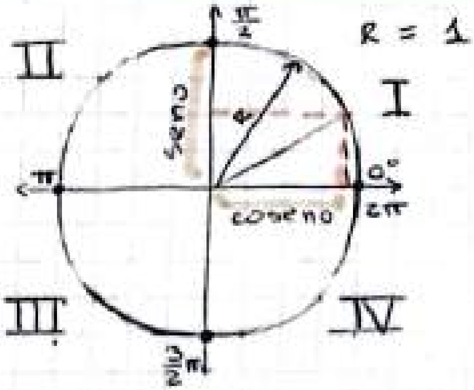
ej =



$$\text{sen } 30° = \frac{x}{50}$$
$$\frac{1}{2} \nearrow \frac{x}{50}$$
$$\boxed{25 = x}$$

$$\cos 30° = \frac{y}{50}$$
$$0.86 = \frac{y}{50}$$
$$43.3 = y$$

CIRCUNFERENCIA TRIGONOMETRICA



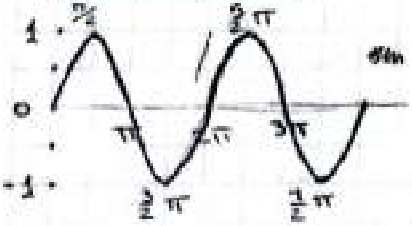
R = 1

TABLA RECORDAR

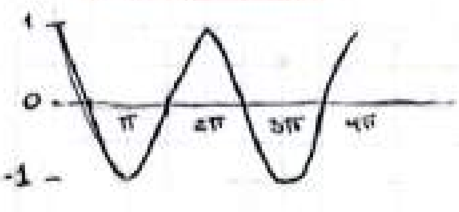
radianes	grados	senos	cosenos
0	0°	0	1
π/6	30°	1/2	√3/2
π/4	45°	√2/2	√2/2
π/3	60°	√3/2	1/2
π/2	90°	1	0
π	180°	0	-1
3/2 π	270°	-1	0
2π	300°	0	1

GRAFICACION GENERAL

sen α = 0



cos α = 1



FORMULAS CAMBIOS

$$\text{grados a radianes} = 1° \cdot \frac{\pi}{180}$$

$$\text{radianes a grados} = 1 \text{ rad} \cdot \frac{180}{\pi}$$

ECUACIONES LOGARITMICAS

DEMOSTRACION

$\log_2 (x+1) = 3$

a b c

$2^3 = x+1$

$8 = x+1$

$\boxed{7 = x}$

$\log_2 (4x) = 2$

si no hay base es 10

$10^2 = 4x$

$100 = 4x$

$\boxed{25 = x}$

$\log_2 (x+1) + \log_2 (x) = 0$

$\log_2 ((x+1)\cdot x) = 0$

$2^0 = x^2 + x$

$1 = x^2 + x \rightarrow 0 = x^2 + x - 1$

$\frac{-1 \pm \sqrt{1+4}}{2} \rightarrow x = \frac{-1-\sqrt{5}}{2}$

$\hookrightarrow x = \frac{-1+\sqrt{5}}{2}$

$\log_2 (x+7) - \log_2 (x+1) = 4$

$\log_2 \frac{x+7}{x+1} = 4$

$2^4 = \frac{x+7}{x+1}$

$16(x+1) = x+7$

$16x + 16 = x + 7$

$15x = -9$

$x = -\frac{9}{15} \rightarrow \boxed{-\frac{3}{5}}$

$\log_2 (4x^2 - 26x + 18) = 5$

$2^5 = 4x^2 - 26x + 18$

$32 = 4x^2 - 26x + 18$

$0 = 4x^2 - 26x - 14$

$0 = 2(2x^2 - 13x - 7)$

$\frac{+13 \pm \sqrt{169+56}}{4}$

$\frac{+13 \pm \sqrt{225}}{4} \rightarrow \frac{13 \pm 15}{4}$ $\begin{cases} x = 7 \\ x = -\frac{1}{2} \end{cases}$

DERIVADAS

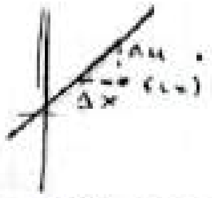
linea recta que cruza dos puntos

↳ la derivada es su pendiente (m/tangente)

como se calcula m

$\boxed{m = \frac{\Delta y}{\Delta x}} = f'(x)$

↳ o h



si coordenada x de 1er punto es x, lo del 2do punto se le suma h

$\Delta y = (f(x) + h) - f(x)$

$y = x^2 \longrightarrow f(x) = x^2$ $\underbrace{f'(x) = 2x}_{\text{funcion derivada}}$

$m = f'(x) = \lim_{\Delta x \to 0} \frac{\Delta x}{\Delta x}$

$\searrow$

$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$\lim_{h \to 0} \frac{(x+h)^2 - x^2}{h} \longrightarrow \lim_{h \to 0} \frac{\cancel{x^2} + h^2 + 2xh - \cancel{x^2}}{h} \rightarrow \lim_{h \to 0} \frac{\cancel{h}(h+2x)}{\cancel{h}}$

$\uparrow$

$\lim_{h \to 0} h + 2x = \boxed{2x}$

EXPONENCIALES Y LOGARITMICAS

PROPIEDADES EXPONENCIALES

$a^2 . \overset{+}{a^3} = a^5$

$a^6 : \overset{-}{a^2} = a^4 \rightarrow a^2 : a^6 = \overset{\text{inversa}}{a^{-4}} = \left(\frac{1}{a}\right)^4 = \frac{1}{a^4}$

$\left(\frac{a}{b}\right)^{-1} = \frac{b}{a} \rightarrow \left(\frac{a}{b}\right)^{-2} = \left(\frac{b}{a}\right)^2 = \frac{b^2}{a^2}$

$a^{-3} = \frac{1}{a^3} \rightarrow \left(\frac{1}{a}\right)^{-3} = a^3$

$a^1 = a \rightarrow a^0 = 1 \rightarrow (a^{\overset{\times}{2}})^3 = a^6$

$(a.b)^2 \neq a^2 + b^2$

$\hookrightarrow a^2 + b^2 + 2.a.b \rightarrow (A+B)(A+B)$

$a^2 + a^2 = 2a^2$

$1\,a^{x+1} + 2\,a^{x+1} = 3 . a^{x+1}$

$a^x + a^{x+1} = a^x + a^x . a^1 \rightarrow a^x(1+a) \rightarrow$ se saca factor común

$\hookrightarrow$ ej $= 2^x + 2^{x+1} = 2^x + 2^x . 2^1 \rightarrow 2^x(1+2)$

PROPIEDADES LOGARITMICAS

$\log_a b = c \iff \boxed{a^c = b}$

$\hookrightarrow$ ej $= \log_2 8 = 3 \iff 2^3 = 8$

$\log_a (x.y) = \log_a x + \log_a y$

$\log_a \left(\frac{x}{y}\right) = \log_a x - \log_a y$

$\log_a x^n = n . \log_a x$

$\log_a b = \frac{\log_c b}{\log_c a} = \frac{\log b}{\log A}$

$\log_a a = 1$

$\log_a 1 = 0$

$\log b = \log_{10} b$

$\boxed{\ln b = \log_e b}$

logaritmo en base e

$\ln_e e = 1$

SUMA RESTA Y MULT X ESCALAR

INTEGRAL DE SUMA O RESTA

$\int [f(x) \pm g(x)] dx = \int f(x) . dx \pm \int g(x) . dx$

EJEMPLO

$\int (x^2 + 2x) \, dx$

$\int x^2 . dx + \int 2x . dx$

INTEGRAL DE FUNCION X UN ESCALAR

$\int k \, f(x) \, dx = k \int f(x) \, dx$

Ejemplo

$\int 2x \, dx$

$2 \int x \, dx =$

Teniendo ambos conocimientos seguimos =

$\int x^2 \, dx + \int 2x \, dx$

$\frac{x^3}{3} + c + 2 \int x \, dx$

$\frac{x^3}{3} + c + \cancel{2} \frac{x^2}{\cancel{2}} + c$

$\frac{x^3}{3} + x^2 + C$ → al valor ser desconocido se suman (+c+c

$\boxed{\frac{1}{3} x^3 + x^2 + C}$ → resolución

$2 \int x \, dx = \cancel{2} \frac{x^2}{\cancel{2}} + c \rightarrow \boxed{x^2 + c}$

INTE DEL PRODUCTO DE SUSTITUCION

INTEGRAL DE MULTIPLICACION

$\int (x+2)(x+1)\,dx =$

$\int (x^2+x+2x+2)\,dx =$

$\int (x^2+3x+2)\,dx$

$\frac{x^3}{3}+\frac{3x^2}{2}+2x+C$

$\boxed{\frac{1}{3}x^3+\frac{3}{2}x^2+2x+C}$

$\int (x^2+2x+3)^3(x+1)\,dx$

$\int u^3(x+1)\frac{du}{2x+2}$

$u = x^2+2x+3$

$du = (2x+2)\,dx$

$\frac{du}{2x+2} = dx$

$\int \frac{u^3(x+1)}{2(x+1)}\,du$

$\int \frac{u^3}{2}\cdot du$

$\frac{1}{2}\int u^3\cdot du$

$\frac{1}{2}\frac{u^4}{4}+C$

$\frac{1}{8}u^4+C = \frac{1}{8}(x^2+2x+3)^4+C$

INTEGRAL POR PARTES

$\int u\cdot dv = u\cdot v - \int v\cdot du$

Aplicaciones

$\int x^2\cdot \ln x\cdot dx$

$u = \ln x$ $\qquad dv = x^2\,dx$

$du = \frac{1}{x}\,dx$ $\qquad v = \frac{x^3}{3}$

$\ln x\cdot x^2\cdot dx$

$\ln x\cdot\frac{x^3}{3}-\int\frac{x^3}{3}\cdot\frac{1}{x}\cdot dx$

$\ln x\cdot\frac{x^3}{3}-\int\frac{x^2}{3}\cdot dx$

$\ln x\cdot\frac{x^3}{3}-\frac{1}{3}\cdot\frac{x}{3}+C$

$\ln x\cdot\frac{x^3}{3}-\frac{1}{9}x^3+C$

$x^3\left(\frac{1}{3}\ln x-\frac{1}{9}\right)+C$

ECUACIONES EXPONENCIALES

DEMOSTRACION

$2^x = 2^3$

informal
$x = 3$

formal
$\log_2 2^x = \log_2 2^3$

$x \cancel{\log_2 2} = 3 \cancel{\log_2 2}$

$\boxed{x = 3}$

$10^x . 10^2 = 10^5$

$10^{x+2} = 10^5$

informal
$x = 3$

formal
$\log_{10} 10^{x+2} = \log_{10} 10^5$

$x+2 \cancel{\log_{10} 10} = 5 \cancel{\log_{10} 10}$

$x + 2 = 5 \rightarrow \boxed{x = 3}$

$9^{x+1} = (\frac{1}{3})^{2x}$

$(3^2)^{x+1} = (3)^{-2x}$

$3^{2x+2} = 3^{-2x}$

$\log_3 3^{2x+2} = \log_3 3^{-2x}$

$2x + 2 \cancel{\log_3 3} = -2x \cancel{\log_3 3}$

$2x + 2 = -2 \rightarrow 4x = -2$

$x = -\frac{1}{2}$

$2 . 3^x + 5 (3^x) - (3^x) = 6$

$(2 + 5 - 1) 3^x = 6$

$6 . 3^x = 6 \rightarrow 3^x = \frac{6}{6}$

$3^x = 1$

informal

formal
$\log_3 3^x = \log_3 1$

$x \cancel{\log_3 3} = 0$

$\boxed{x = 0}$

$5^{2x} = 5^6$

informal
$x = 3$

formal
$\log_5 5^{2x} = \log_5 5^6$

$2x \cancel{\log_5 5} = 6 \cancel{\log_5 5}$

$2x = 6 \rightarrow \boxed{x = 3}$

$2^{x+8} = 32$

$\log_2 2^{x+8} = \log_2 32$

$x + 8 \cancel{\log_2 2} = 5$

$x + 8 = 5 \rightarrow \boxed{x = -3}$

$8^{(x^2 + 3x + 2)} = 1$

$x^2 + 3x + 2$
A B C

$\frac{-3 \pm \sqrt{9-8}}{2} \rightarrow \frac{-3 \pm 1}{2}$ $x =$ x

$2^{x+3} + 2^{x-2} - 2^{x+1} = 50$

$2^x . 2^3 + 2^x 2^{-2} - 2^x . + 2^1 = 50$

$2^x . 8 + 2^x . \frac{1}{4} - 2^x . 2 = 50$

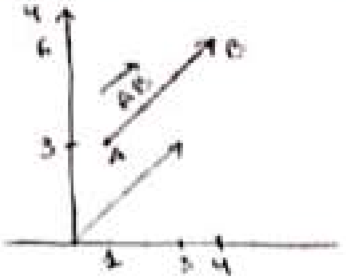
$(8 + \frac{1}{4} - 2) . 2^x = 50$

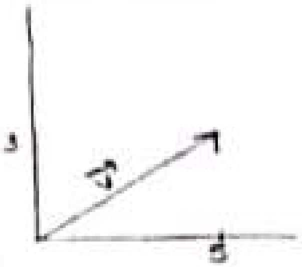
$(\frac{6}{1} + \frac{1}{4}) 2^x = 50 \rightarrow (\frac{25}{4}) 2^x$

$2^x = \frac{50 . 4}{25} \rightarrow 2^x = 8$

informal
$x = 3$

ESCRITURA Y COORDENADAS





CALCULO

$\vec{AB} = B - A$

$\vec{AB} = (4;6) - (1;3)$

$\vec{AB} = (3;3)$

CALCULO

$\vec{v} = (5;3)$

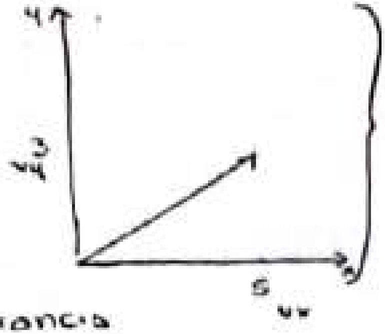
MODULO

$|\vec{v}| = \sqrt{vx^2 + vy^2}$

$|\vec{v}| = \sqrt{5^2 + 3^2}$

$|\vec{v}| = \sqrt{25-9}$

$|\vec{v}| = \sqrt{34}$ → distancia



como tiene 2 componentes es R2

$|\vec{u}| = (5;3;-1)$

$|\vec{u}| = \sqrt{(ux^2 + uy^2 + uz^2}$

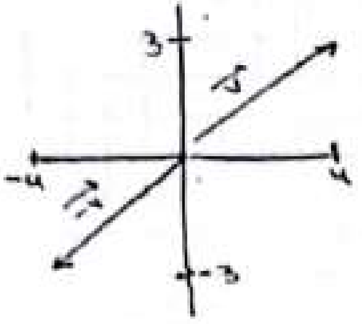
$|\vec{u}| = \sqrt{(5^2 + 3^2 + (-1)^2}$

$|\vec{u}| = \sqrt{25 + 9 + 1} \rightarrow |\vec{u}| = \sqrt{35}$

como tiene 3 componente es R^3

VECTOR OPUESTO

aquel que va en sentido contrario pero respetando misma dirección del vector original



$\vec{v} = (4;3)$

$-\vec{v} = (-4;-3)$

signos cambian

DIVISION DE DERIVADAS

$f(x) = \frac{g(x)}{h(x)} \rightarrow f'(x) \frac{g'(x) \cdot h(x) - g(x) \cdot h'(x)}{h(x)^2}$

ejemplos:

$f(x) = \frac{2x}{x+1}$

$f'(x) = \frac{2(x+1) - 2x(1+0)}{(x+1)^2}$

$f'(x) = \frac{2x+2-2x}{(x+1)^2}$

$\boxed{f'(x) = \frac{2}{(x+1)^2}}$

REGLA DE LA CADENA

→ para funciones compuestas

$f \circ g(x) = f(g(x))$

$f'(x) = f'(g(x)) \cdot g(x)$

ejemplos

$f(x) = (2x+3)^2$

$f(x)\ x^2$

$g(x) = 2x+3$

$f'(x) = 2(2x+3)^2 \cdot (2+0)$

$f'(x) = 4(2x+3)$

$\boxed{f'(x) = 8x+12}$

$f(x) = \ln(3x+4)$

$f(x) = \ln x$

$g(x) = 3x+4$

$f'(x)\ \frac{1}{3x+4} \cdot (3+0)$

$\boxed{f'(x) = \frac{3}{3x+4}}$

$f(x) = e^{3x^2+1}$

$f(x)\ e^x$

$g(x) = 3x^2+1$

$f'(x) = e^{3x^2+1} \cdot (6x+0)$

$\boxed{f'(x) - e^{3x^2+1} \cdot 6x}$

$f(x) = \text{sen}\left(\frac{5x}{x^2}\right)$

$f(x) = \text{sen}\ x$

$g(x) = \frac{5x}{x^2}$

$f'(x) = \cos\left(\frac{5x}{x^2}\right) \cdot \frac{5x^2 - 10x^2}{x^4}$

$f'(x) = \cos\left(\frac{5\cancel{x}}{x^{\cancel{2}}}\right)\left(\frac{-5\cancel{x^2}}{x^{\cancel{4}}}\right)$

$\boxed{f'(x) = \frac{-5\cos(5/x)}{x^2}}$

ECUACIONES TRIGONOMETRICAS

PRACTICA / DEMOSTRACIÓN $[0, 2\pi] \rightarrow 0 \leq x \leq 2\pi$

$4 \operatorname{sen}\left(x + \frac{\pi}{2}\right) = 2$

$\operatorname{sen}\left(x + \frac{\pi}{2}\right) = \frac{2}{4}$

$\underbrace{\operatorname{sen}\left(x + \frac{\pi}{2}\right)}_{} = \frac{1}{2}$

$\rightarrow \frac{1}{6}\pi$ (30°)

$\rightarrow \frac{5}{6}\pi$ (150°)

$x + \frac{\pi}{2} = \frac{1}{6}\pi$

$x = \frac{1}{6}\pi - \frac{1}{2}\pi$

$x = \frac{1-3}{6}\pi$

$x = -\frac{2}{6}\pi \rightarrow \boxed{-\frac{1}{3}\pi}$

$x + \frac{\pi}{2} = \frac{5}{6}\pi$

$x = \frac{5}{6}\pi - \frac{1}{2}\pi$

$x = \frac{5-3}{6}\pi$

$x = \frac{2}{6}\pi \rightarrow \boxed{\frac{1}{3}\pi}$

$3\cos\left(2x - \frac{3}{2}\pi\right) = -3$

$\cos\underbrace{\left(2x - \frac{3}{2}\pi\right)}_{} = \underbrace{-1}_{\pi}$

$2x - \frac{3}{2}\pi = \pi$

$2x = 1\pi + \frac{3}{2}\pi$

$2x = \frac{5}{2}\pi$

$x = \frac{5}{2} : \frac{2}{1}\pi$

$\boxed{x = \frac{5}{4}\pi}$

IDENTIDADES TRIGONOMETRICAS

$\operatorname{sen}\alpha = \frac{op}{hip} \longrightarrow \operatorname{cosec}\alpha = \frac{1}{\operatorname{sen}\alpha} \longrightarrow \operatorname{cosec} = \frac{hip}{op}$ (inversa del seno)

$\cos\alpha = \frac{ady}{hip} \longrightarrow \sec\alpha = \frac{1}{\cos\alpha} \longrightarrow \sec\alpha = \frac{hip}{ady}$ (inversa del coseno)

$\tan\alpha = \frac{op}{ady} \longrightarrow \operatorname{cotan}\alpha = \frac{1}{\tan\alpha} \longrightarrow \operatorname{coton}\alpha = \frac{ady}{op}$ (inverso del tangente)

$\tan\alpha = \frac{\operatorname{sen}\alpha}{\cos\alpha} \rightarrow \operatorname{coton}\alpha = \frac{\cos\alpha}{\operatorname{sen}\alpha}$

$\operatorname{sen}^2\alpha + \cos^2\alpha = 1$

$\downarrow$

$\operatorname{sen}^2\alpha = 1 - \cos^2\alpha$ o $\cos^2\alpha = 1 - \operatorname{sen}^2\alpha$

$\downarrow$ $\downarrow$

$\operatorname{cosec}^2\alpha = 1 + \operatorname{cotan}^2\alpha$ $\sec^2\alpha = 1 + \tan^2\alpha$

$\operatorname{sen}(-\alpha) = -\operatorname{sen}(\alpha)$ $\cos(-\alpha) = -\operatorname{sen}(\alpha)$ $\tan(-\alpha) = -\tan\alpha$

$\operatorname{sen}\left(\overset{\beta}{\frac{\pi}{2} - \alpha}\right) = \cos\alpha$ $\cos\left(\overset{\beta}{\frac{\pi}{2} - \alpha}\right) = \operatorname{sen}\alpha$ $\tan\left(\overset{\beta}{\frac{\pi}{2} - \alpha}\right) = \operatorname{coton}\alpha$

$\alpha + \beta = \boxed{\frac{\pi}{2}} \rightarrow$ complementarios

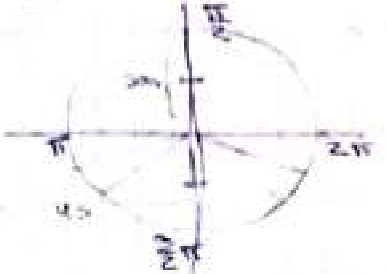
RAICES O CONJUNTO DE 0

DEMOSTRACIÓN

$F(x) = -2\,sen\left(2x + \frac{3}{2}\pi\right) - 1 \rightarrow \frac{2\pi}{2} = \boxed{\pi}$ periodo

$0 = -2\,sen\left(2x + \frac{3}{2}\pi\right) - 1$

$-\frac{1}{2} = sen \underbrace{\left(2x + \frac{3}{2}\pi\right)}_{\alpha}$



$2x + \frac{3}{2}\pi = -\frac{\pi}{6}$ o $2x + \frac{3}{2}\pi = \frac{7}{6}\pi$

$2x = -\frac{1}{6}\pi - \frac{9}{6}\pi$ $\quad 2x = \frac{7}{6} - \frac{9}{6}\pi$

$2x = -\frac{10}{6} \rightarrow -\frac{5}{3}\pi$ $\quad 2x = -\frac{2}{6}\pi \rightarrow -\frac{1}{3}\pi$

$\cancel{2x} = -\frac{5}{3} \times \frac{2}{1}\pi$ $\quad x = -\frac{1}{3} : \frac{2}{1}$

$\boxed{x = -\frac{5}{6}\pi}$ $\quad \boxed{x = -\frac{1}{6}}$

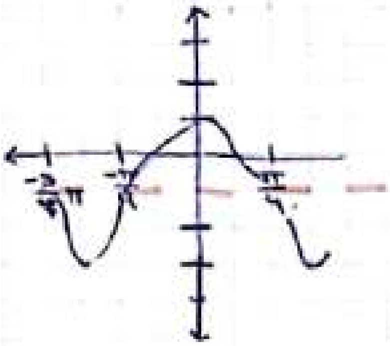
ANALISIS DE FUNCION TRIGONOMETRICA

$\boxed{K \in Z}$

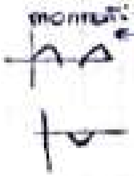
$f(x) = -2\,sen\left(2x + \frac{3}{2}\pi\right) - 1$ $\quad$ Dom = $\mathbb{R}$

$\downarrow$ $-\frac{2}{2}$ $\quad \left(2\left(x + \frac{3}{2}\pi\right)\right)$ $\quad \downarrow$

Rango / imagen: $[-3, 1]$



$C^0 = \left\{-\frac{5}{6}\pi + \pi k \,;\, \cdot\frac{1}{6}\pi + \pi k\right\}$



$C^+ = \left(-\frac{\pi}{6} + \pi k \,;\, \frac{\pi}{6} + \pi k\right)$

$C^- = \left(-\frac{5}{6}\pi + \pi k \,;\, -\frac{1}{6}\pi + \pi k\right)$

ORDENADA AL ORIGEN

$f(0) = -2\,sen\left(2.0 + \frac{3}{2}\pi\right) - 1$

$f(0) = -2\,sen\left(\frac{3}{2}\pi\right) - 1$

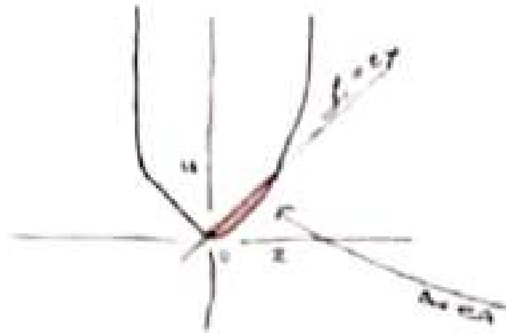
$f(0) = -2 . (-1) - 1$

$f(0) = 2 - 1$

$\boxed{f(0) = 1}$

Area entre dos funciones

$f = (x^2)$



$$\int_0^2 (2x - x^2)\,dx$$

$$\frac{2x^2}{2} - \frac{x^3}{3}\Big]_0^2$$

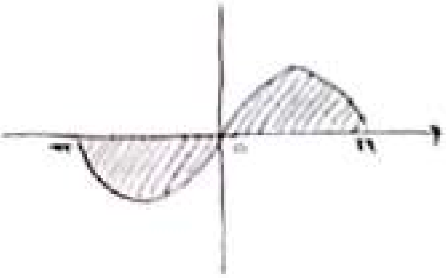
$$\left[x^2 - \frac{x^3}{3}\right]_0^2 \to \left(2^2 - \frac{2^3}{3}\right) - \left(0^2 - \frac{0^3}{3}\right)$$

$$4 - \frac{8}{3} - (0 - 0)$$

$$\frac{12 - 8}{3}$$

$$\boxed{\frac{4}{3}}$$

Area entre 2 funciones con cruce con eje x



$f(x) = \text{sen}\, x$

$g(x) = 0$

$$A = \int_{-\pi}^{0} 0 - \text{sen}\, x + \int_0^{\pi} \text{sen}\, x - 0$$

$$A = \cos x\Big]_{-\pi}^{0} + -\cos x\Big]_{\pi}^{0}$$

$$A = \cos 0 - \cos(-\pi) + -\cos \pi - (-\cos 0)$$

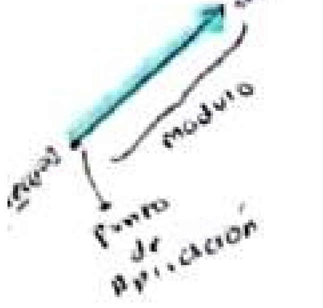
$$A = 1 + 1 + \cancel{1} + 1 + 1$$

$$A = 2 + 2$$

$$\boxed{A = 4}$$

VECTORES

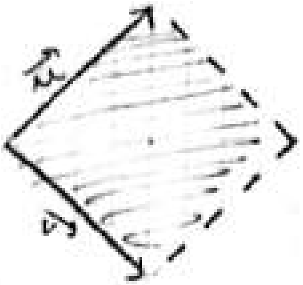
→ son flechas



- modulo = tamaño
- sentido = indica hacia donde va
- direccion = recta de accion donde actua el vector
  - └→ inclinacion de vector

PRODUCTO VECTORIAL → AREA

$$\left.\begin{array}{l}\vec{u} = (1; -2; 2) \\ \vec{v} = (-1, 4; 1)\end{array}\right] \vec{u} \cdot \vec{v} = \begin{array}{ccc} i & -j & k \\ 1 & -2 & 2 \\ -1 & 4 & 1 \end{array}$$



$$\vec{u} \cdot \vec{v} = (\overset{i}{-2} - 8; \overset{-j}{1 + 2}, \overset{k}{4} - 2)$$

$$\vec{u} \cdot \vec{v} = \underbrace{(-10; -3; 2)}_{\text{vector}}$$

Area =

$$|\vec{u} \times \vec{v}| = \sqrt{(-10)^2 + (-3)^2 + (2)^2}$$

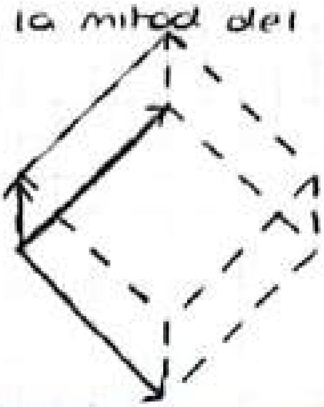
$$|\vec{u} \times \vec{v}| = \sqrt{100 + 9 + 4}$$

$$\underbrace{|\vec{u} \times \vec{v}| = \sqrt{113}}_{\text{Area paralelogramo}} \longrightarrow \underbrace{\frac{\sqrt{113}}{2}}_{\text{Area triangulo}}$$

El Area del triangulo es la mitad del paralelogramo

PRODUCTO MIXTO

$$\vec{u} = (1; -2; 2)$$
$$\vec{v} = (-1; 3; 1)$$
$$\underbrace{\vec{w} = (2; 1; 2)}$$
$$\downarrow$$



PARALELEPIPIDO

$$\vec{u} \cdot \vec{v} \cdot \vec{w} = \begin{array}{ccc} 1 & -2 & 2 \\ -1 & 3 & 1 \\ 2 & 1 & 2 \end{array}$$

$$\vec{u} \cdot \vec{v} \cdot \vec{w} = (1.((3.2) - (1.1)) + 2((-1.3) - (2.1)) + 2((-1.1) - ($$

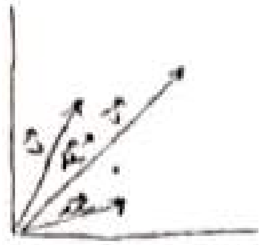
$$\vec{u} \cdot \vec{v} \cdot \vec{w} = (1.(6 - 1) + 2(-2, -2) + 2(-1 - 6)$$

$$\vec{u} \cdot \vec{v} \cdot \vec{w} = (5) + (8) - (14)$$

$$|\vec{u} \cdot \vec{v} \cdot \vec{w}| = |-17|$$

$$\underbrace{|\vec{u} \cdot \vec{v} \cdot \vec{w}| = 17}_{\text{Volumen del paralelepipedo}}$$

SUMA DE VECTORES



$$+\ \begin{array}{l} \vec{u} = (3,2) \\ \vec{v} = (2,6) \\ \hline \vec{u}+\vec{v} = (5;8) \end{array}$$

R^2

$$+\ \begin{array}{l} \vec{u} = (3;-2;8) \\ \vec{v} = (-2,0,-5) \\ \hline \vec{u}+\vec{v} = (1;2;3) \end{array}$$

R^3

RESTA DE VECTORES

$$-\ \begin{array}{l} \vec{u} = (-2;5) \\ \vec{v}\ \ (3;-4) \\ \hline \vec{u}-\vec{v} = (-5;9) \end{array} \rightarrow -\vec{v} = (-3,4)$$

$$+\ \begin{array}{l} \vec{u} = (-2;5) \\ -\vec{v} = (-3;4) \\ \hline \vec{u}-\vec{v} = (-5,9) \end{array}$$

PRODUCTO POR ESCALAR / MULTIPLICACION

$\vec{v} = (2;5;-3)$

$3.\vec{v} = 3\,(2;5;-3)$

$3.\vec{v} = (6;15;-9)$

PRODUCTO ESCALAR

$$\left.\begin{array}{l} \vec{u} = (2;-5) \\ \vec{v} = (-1;4) \end{array}\right] \vec{u}.\vec{v} = 2(-1)+(-5).4 \rightarrow -2-20 = \boxed{-22}$$

$$\vec{u}.\vec{v} \overset{\Downarrow}{=} -22$$

Si producto escalar da 0 los vectores son perpendiculares

PRODUCTO VECTORIAL (R^3)

$$\left.\begin{array}{l} \vec{u} = (1,2,3) \\ \vec{v} = (-3;4;-1) \end{array}\right\} \vec{u}.\vec{v} = \begin{array}{ccc} i & j & k \\ 1 & 2 & 3 \\ -3 & 4 & -1 \end{array}$$

$$\vec{u}.\vec{v} = (\underbrace{2.(-1) - 4.3}_{i}; \underbrace{1(-1) - (-3)3}_{-j}; \underbrace{1.4 - (-3).2}_{k})$$

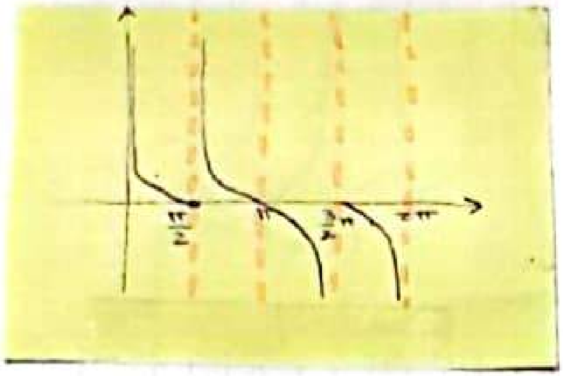
$\vec{u}.\vec{v} = (-2-12; -1+9; 4+6)$

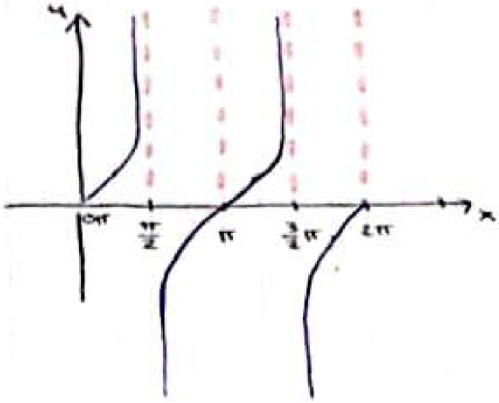
$\vec{u}.\vec{v} = (-14; 8; 10)$

$\vec{u}.\vec{v} = (-14;-8;10)$

FUNCION TANGENTE

$f(x) = -\tan(x)$ $[0, 2\pi]$ $f(x) = \tan(x)$





$f(x) = a \tan(b(x+c)) + d$

- D: eleva la coordenada de x ±
- A: cambia curvatura + sea menor curva
- C: es donde inicia donde es completo
- B = per = $\frac{2\pi}{B}$

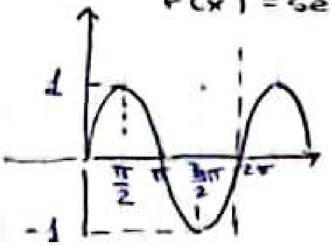
ejemplo = $f(x) - 2\tan\underbrace{(2x - \pi)}_{2(x) - \boxed{\frac{\pi}{2}}} + 1$

$\frac{\pi}{2}$ → inicio

$\frac{2\pi}{2} = \boxed{\pi}$ → periodo

¿Qué es 2kπ?

$f(x) = \text{sen}(x)$



Hallar x / f(x) = 1 → $\frac{\pi}{2}$, $\frac{5}{2}\pi$ $\Big\}$ $x = \left\{\frac{\pi}{2} + \overset{\text{periodo . K}}{(2\pi) \cdot K}\right\}$

$K \in \boxed{Z}$ → numeros enteros

Hallar x / f(x) = 0 → π, 2π, 0 $\Big\}$ $x = \{0 + 2\pi \cdot K ; \pi + 2\pi \cdot K\}$

$K \in Z$

FORMATO INFINITO

INTEGRALES

QUE ES?

$$f(x) \rightarrow f'(x) \rightarrow \int f'(x) = f(x)$$

la integral de una derivada es la funcion original

$$f(x) \rightarrow f'(x) \rightarrow f''(x) \rightarrow \int f''(x) = f'(x)$$

la integral es el volver al paso anterior de derivar

ejemplo

$\int f(x) . dx$

$f'(x) = 2x$

$\frac{d.f}{dx}(x)$

$\int 2x . dx = \overset{\text{anterior a derivada}}{x^2} + C$ → se agregan cuando integrales son indefinidas

se pasa x a

+C da a los primitivas

TABLA DE INTEGRALES

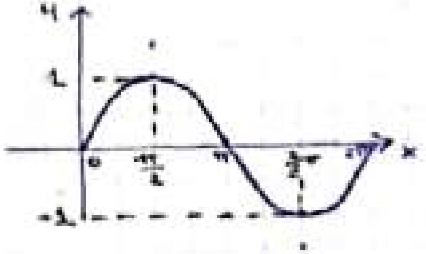
→ C = posible valor cualquiera cero o real

DERIVADA	INTEGRALES
$f'(x) = 0$	
$f'(x) = 1$	$\int 1 . dx = x + C$
$f'(x) = n . x^{n-1}$	$\int x^n dx = \frac{x^{n+1}}{n+1} + C$ → $f(x) = x^2 \rightarrow f'(x) = 2x \rightarrow \int 2x . dx = \frac{\cancel{2}x^2}{\cancel{2}}$
$f'(x) = \frac{1}{x}$	$\int \frac{1}{x} dx = \ln\lvert x\rvert + C$
$f'(x) = \frac{1}{x} \log_a e$	$\int \frac{1}{x} \log_a e . dx = \log_a x + C$
$f'(x) = e^x$	$\int e^x . dx = e^x + C$
$f'(x) = a^x . \ln a$	$\int a^x . \ln a . dx = a^x + C$
$f'(x) = \cos x$	$\int \cos x . dx = \operatorname{sen} x + C$
$f'(x) = -\operatorname{sen} x$	$\int \operatorname{sen} x . dx = -\cos x + C$

FUNCIONES TRIGONOMETRICAS

FUNCION SENO

$f(x) = \text{sen}(x)$ → $f(x) = a\ \text{sen}(b(x+c)) + d$



↳ $|a|$ = amplitud simetria ([illegible])

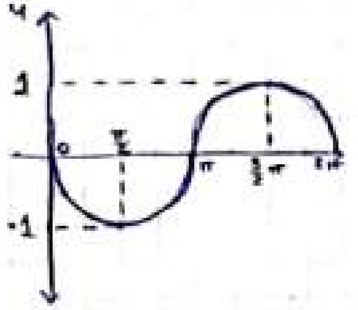
↳ d = desplazamiento vertical ↑↓

Rango (imagen) = $(d - |a| ; d + |a|)$
$\quad\downarrow$ f. min $\quad\downarrow$ f. max

↳ b = frecuencia ~~[illegible]~~
periodo = $\frac{2\pi}{b}$ → lo que tarda hacer 1 vuelta — 1 vuelta

↳ c = desfasaje ⇄
↓ + = ← ↓ − = →

$f(x) = -\text{sen}(x)$



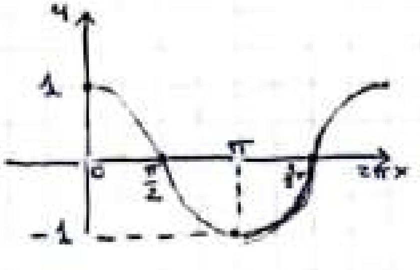
↳ $b(x+c)$ = angulo de fase
↙ α + pe ↓ final
↓ se inicio

¿PARA QUE?

FUNCIÓN COSENO

$f(x) = \cos(x)$ → $f(x) = a\ \cos(b(x+c)) + d$

periodo = $\frac{2\pi}{b}$ → 1 vuelta

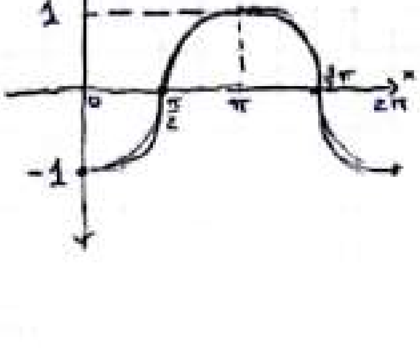


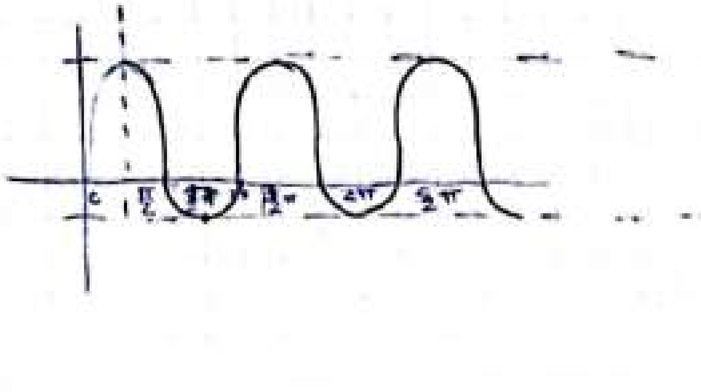
ejemplo = $f(x) = \underset{a}{2}\cos(\underset{b}{2}x - \underset{c}{\pi}) + \underset{d}{1}$

$2(x - \underset{c}{\frac{\pi}{2}})$

periodo $\frac{2\pi}{2} = \pi$

$f(x) = -\cos(x)$





si $f'(1) = 2 \cdot 1$

$\boxed{f'(1) = 2}$ → es la pendiente de la recta tangente a $y = x^2$ en $x = 1$

TABLA DE DERIVADAS

$f(x) = k \rightarrow f'(x) = 0$ → cualquier real si lo derivo me da 0

$f(x) = x \rightarrow f'(x) = 1$ → si $f(x) = 3x \rightarrow f'(x) = 2$

$f(x) = x^n \rightarrow f'(x) = n\,x^{n-1}$ → exponente baja a multiplicar y en exp dismi.

$f(x) = \ln x \rightarrow f'(x) = \frac{1}{x}$

$f(x) = \log_a x \rightarrow f'(x) = \frac{1}{x} \log_a e$

$f(x) = e^x \rightarrow f'(x) = e^x$ → su derivada vale lo mismo

$f(x) = a^x \rightarrow f'(x) = a^x . \ln a$

$f(x) = \text{sen}\, x \rightarrow f'(x) = \cos x$

$f(x) = \cos x \rightarrow f'(x) = -\text{sen}\, x$

ALGEBRA DE DERIVADAS

SUMA DE DERIVADAS

$f(x) = g(x) + h(x) \rightarrow f'(x) = g'(x) + h'(x)$

ejemplos:

$f(x) = 2x + 3$

$f'(x) = 2 + 0$

$\boxed{f'(x) = 2}$

$f(x) = 3x^2 + 2x + 1$

$f'(x) = 6x + 2 + 0$

$\boxed{f'(x) = 6x + 2}$

MULTIPLICACION DE DERIVADAS

$f(x) = g(x) . h(x) \rightarrow f'(x) = g'(x) . h(x) + g(x) . h'(x)$

ejemplos =

$f(x) = x\,(x+2) \longrightarrow f(x) = x^2 + 2x$

$g'(x) = 1\,(x+2) + x\,(1+0)$

$f'(x) = x + 2 + x$

$\boxed{f'(x) = 2x + 2}$

$\boxed{f'(x) = 2x + 2}$

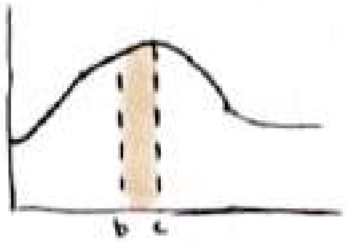
$f(x) = k .$

$f'(x) = k$

INTEGRAL DEFINIDA: AREA BAJO CURVA

integral $\int f(x)\,dx$

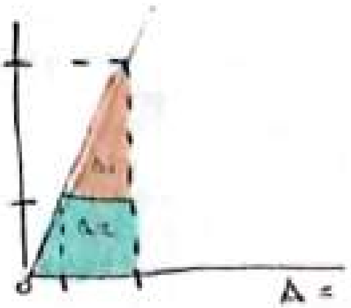
es el area debajo de la curva entre los puntos



si $f(x^2) \to f'(2x) \to \int f(x^2)$

a $\int_b^c f(x)\,dx$

ejemplos



$\int_0^3 2x\,dx$

$2\int_0^3 x \cdot dx$

$\left[\cancel{2}\left(\frac{x^2}{\cancel{2}}\right)\right]_0^3$

$[x^2]_0^3 = 3^2 - 0^2 = 9 \to$ comprobar

$A = \frac{b \cdot h}{2}$

$A = \frac{3 \cdot 6}{2}$

$A = 9$

$\int_1^3 2x \cdot dx$

$2\int_1^3 x \cdot dx$

$\left[\cancel{2}\left(\frac{x^2}{\cancel{2}}\right)\right]_1^3$

$[x^2]_1^3 = 3^2 - 1^2 = \boxed{8}$

comprobar

$A = A_1 + A_2$

$A = \frac{2 \cdot 4}{2} + 2^2$

$A = 4 + 4 \to \boxed{8}$

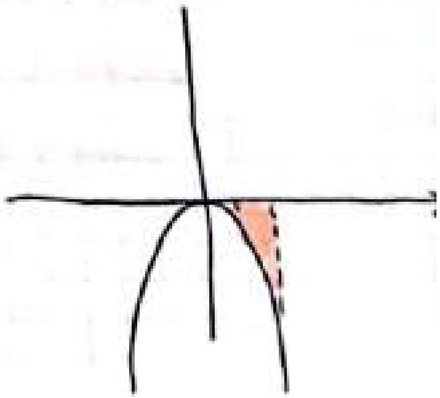
AREA BAJO CURVA DEBAJO DEL EJE X

$\int_1^3 -x^2 \cdot dx$

$(-1)\int_1^3 x^2\,dx$

$\left[(-1)\,\frac{x^3}{3}\right]_1^3 = -1 \cdot \left(\frac{3^3}{3} - \frac{1^3}{3}\right)$

$-1 \cdot \frac{26}{3} \to \boxed{-\frac{26}{3}}$



APLICACION DE DERIVADAS

RECTA TANGENTE

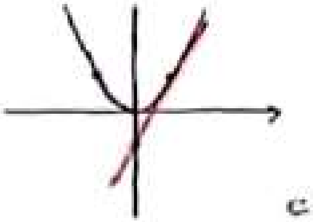
$\rightarrow y = mx + b$

$y = f(x) . x + b$

$f'(x) = \frac{\text{vertical}}{\text{horizontal}} \rightarrow$ pendiente

↓ derivada es la pendiente de una recta

$\downarrow \frac{\Delta y}{\Delta x} \rightarrow \boxed{m}$



$Ej = f(x) = x^2$

$f'(x) = 2x$

$f'(1) = 2.1 \rightarrow 2$

encontremos que

$y = 2x + b$

↓

aprovechamos info que tenemos para sacar b → punto (1,

$1 = 2\,1 + b \leftarrow$

$1 - 2 = b$

$\boxed{-1 = b}$

$y = 2x - 1 \leftarrow$

MAXIMOS Y MINIMOS (RELATIVOS)

EJEMPLOS

$f(x) = x^3 - 6x^2 + 9x - 4 \longrightarrow f(x) = (x-1)^2 (x-4) \begin{cases} \rightarrow 1 \\ \rightarrow 4 \end{cases}$

$f'(x) = 3x^2 - 12x + 9$

$f'(x) = 3(x^2 - 4x + 3)$

$\frac{4 \pm \sqrt{16 - 12}}{2}$

$\frac{4 \pm \sqrt{4}}{2} \rightarrow \frac{4 \pm 2}{2} \begin{cases} 3 \\ 1 \end{cases}$

$x \rightarrow$	$(-\infty, 1)$	1	$(1;3)$	3	$(3; +\infty)$
$f'(x) \rightarrow$	+	0 máx	−	0 mín	+

se reemplaza x=0 en derivada
$f'(x) = 3(x^2 - 4x + 3)$
$0 - 0 + 3$

se reemplaza x=2 en derivada
$f'(x) = 3(x^2 - 4x + 3)$
$4 - 8 + 3$

se reemp
x=4 en
$f'(x) = 3(x^2$
16

$f(1) = (1-1)^2 (1-4)$

$\boxed{f(1) = 0}$

$f(3) = (3-1)^2 (3-4)$

$\boxed{f = (3) = 4}$

$C^{+} = (\pm\infty; 1) \cup (3; +\infty)$

$C^{-} = (1; 3)$

Max $(1, 0)$

Min $(3; -4)$